Virtual Worlds as Fuzzy Cognitive Maps

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Abstract

Fuzzy cognitive maps (FCMs) can structure virtual worlds. FCMs link causal events, values, goals, and trends in a fuzzy feedback dynamical system. They direct actors in virtual worlds as the actors react to events and to one another. In nested FCMs each causal concept can control its own FCM. This combines levels of fuzzy systems that can choose goals or move objects. Adaptive FCMs change as causal patterns change. They adapt with differential Hebbian learning. We apply FCMs to an undersea virtual world of dolphins.

I. Fuzzy Virtual Worlds

Virtual worlds show how actors relate to one another and to their physical or social environments [4,10]. Events cause one another to some degree. Events occur and concepts hold only to some degree. In this sense virtual worlds are fuzzy causal worlds.

Fuzzy cognitive maps (FCMs) show how causal concepts affect one another to some degree [7,8]. A time-varying causal concept $C_i(t)$ measures the degree that some fuzzy event occurs. Causal concepts in a virtual world include events, values, moods, trends, or goals. The causal edges link the concepts to some degree in a feedback network. FCMs model the virtual world as a group of fuzzy classes and causal relations between classes.

FCMs can form the basis for a virtual world. Nodes can stand for actors or for parts of the environment. Variable edges can model the virtual worlds' causal dynamics. FCMs have modeled political events and processes [9,11]. FCM flow may also help visualize dynamical systems.

FCMs combine additively. The new FCM is the union of all the causal concepts in the combined FCMs. This makes it easy to add or delete actors or change the environment in a virtual world or to combine virtual worlds.

The FCM itself acts as a nonlinear neural dynamical system. The FCM learns or adapts when the causal edges change with neural learning laws. FCM feedback dynamics differ in kind from the graph search techniques used to search decision trees and expert systems.

2. Fuzzy Cognitive Maps

FCMs are signed digraphs with feedback [5,6]. Nodes stand for causal fuzzy sets where events occur to some degree. Edges stand for causal flow between the concepts. The sign of an edge (+ or -) stands for causal increase or decrease between nodes. A FCM state is a state vector in $I^n = [0,1]^n$, the fuzzy hypercube. The FCM itself is a nonlinear dynamical system that resembles a neural network and digs a trajectory in I^n. Input states perturb the FCM dynamics and the system quickly converges or "settles down" to an equilibrium attractor, usually a limit cycle or fixed point.

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Simple or trivalent FCMs have causal edge weights in the set {-1,0,1} and concept values in {0,1}. We can draw FCMs from scientific articles, editorials or expert surveys. Most people can state the sign of a causal edge. The hard part is to state its degree or magnitude. We can also average expert responses[12].

Causal concepts or fuzzy sets can stand for actions, events, values, or goals. Each causal node is a nonlinear function that maps the input causal activation into an output fuzzy degree. Figure 1 shows a FCM for a virtual dolphin world. It gives the causal links between the goals and actions in the life of a dolphin [13]. The adjacency or edge matrix $\mathbf{E_D}$ states these causal links in numbers.

$$\mathbf{E}_D = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \\ D_9 \\ D_{10} \end{array} \begin{array}{c} \begin{array}{cccccccccc} D_1 & D_2 & D_3 & D_4 & D_5 & D_6 & D_7 & D_8 & D_9 & D_{10} \end{array} \\ \left[\begin{array}{cccccccccc} 0 & -1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 & -1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 1 & -1 & -1 & -1 & 0 & 1 \\ -1 & -1 & 1 & 0 & -1 & -1 & -1 & -1 & -1 & 0 \end{array}\right] \end{array}$$

where the ith row lists the connection strength of the edges e_{ik} directed out from causal concept C_i and the ith column lists the edges e_{ki} directed into C_i. C_i causally increases C_k if $e_{ik} > 0$, decreases C_k if $e_{ik} < 0$, and has no effect if $e_{ik} = 0$.

FCMs recall as the FCM dynamical system equilibrates. Simple FCM inference is

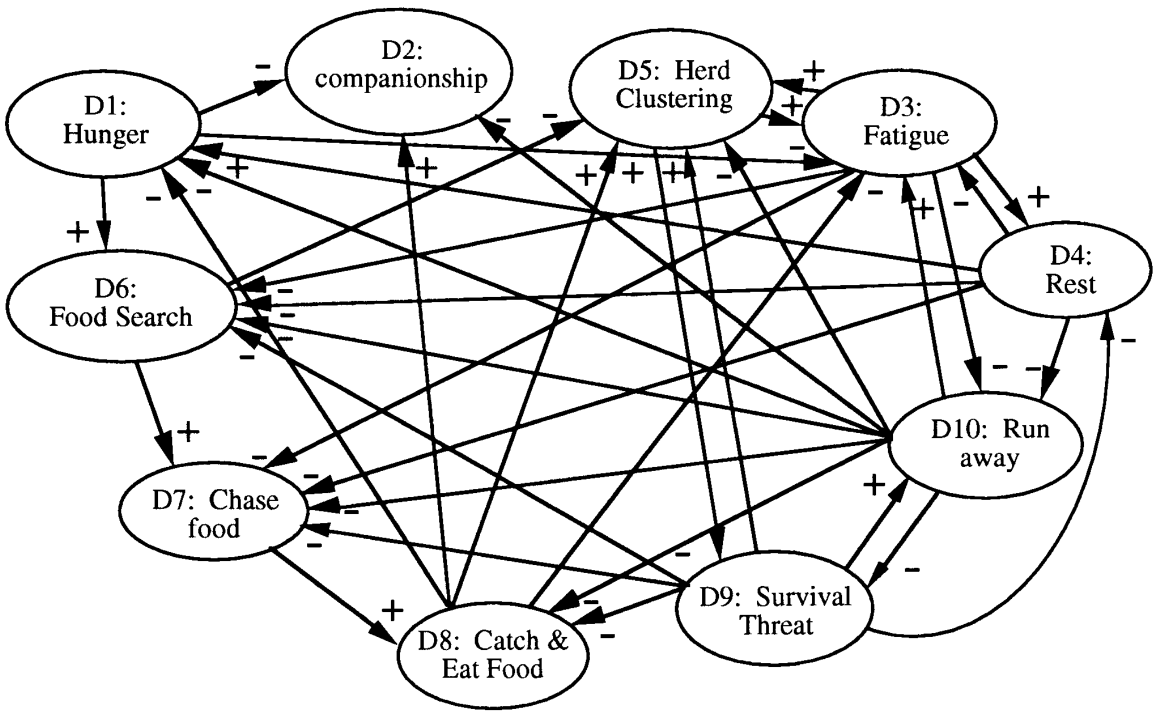

Figure 1. Trivalent fuzzy cognitive map for the control of a dolphin actor in a fuzzy virtual world. The edges connect causal concepts in a signed adjacency matrix.

matrix-vector multiplication followed by thresholding[8,9]. State vectors $\mathbf{C}_n$ cycle through the FCM adjacency matrix **E**. The system nonlinearly transforms the result of each pass into a fuzzy value at each node:

$$C_i(t_{n+1}) = S\left[\sum_{k=1}^{N} e_{ki}(t_n) C_k(t_n)\right] \tag{1}$$

where $S(x)$ is a bounded signal function.

Simple threshold FCMs quickly converge to stable limit cycles or fixed points [8,9]. These limit cycles show "hidden patterns" in the causal flow through the FCM. These patterns establish the rhythm of the world.

We can model the effect of a survival threat on the dolphin FCM shown in Figure 1 as a sustained input to D_9. $\mathbf{C}_0$ is the initial input vector to the dolphin FCM:

$\mathbf{C}_0 = [\,0\,0\,0\,0\,0\,0\,0\,0\,1\,0]$.

Then

$\mathbf{C}_0\,\mathbf{E}_D = [\,0\,0\,0\,\text{-}1\,1\,\text{-}1\,\text{-}1\,0\,0\,1] \rightarrow \mathbf{C}_1 = [\,0\;0\;0\;0\;1\;0\;0\;0\;1\;1]$.

The arrow denotes a threshold operation with 1/2 as the threshold value. $\mathbf{C}_1$ keeps D_9 on (D_9=1) since we want to study the effect of a sustained threat. When threatened the dolphins cluster in a herd and flee the threat ($\mathbf{C}_1$). The negative weights in the FCM show that a threat to survival turns off all other actions. If the threat persists, the limit cycle $\mathbf{C}_1 \rightarrow \mathbf{C}_2 \rightarrow \mathbf{C}_3 \rightarrow \mathbf{C}_4 \rightarrow \mathbf{C}_1 \ldots$ appears.

$\mathbf{C}_1\,\mathbf{E}_D = [\,2\;\text{-}1\;2\;0\;0\;\text{-}2\;\text{-}2\;\text{-}1\;\text{-}2\;1] \rightarrow \mathbf{C}_2 = [\,0\;0\;1\;0\;0\;0\;0\;0\;1\;1]$,
$\mathbf{C}_2\,\mathbf{E}_D = [\,2\;\text{-}2\;0\;1\;1\;\text{-}2\;\text{-}3\;\text{-}1\;\text{-}1\;0] \rightarrow \mathbf{C}_3 = [\,0\;0\;0\;1\;1\;0\;0\;0\;1\;0]$,
$\mathbf{C}_3\,\mathbf{E}_D = [\,2\;\text{-}1\;\text{-}1\;\text{-}1\;2\;\text{-}1\;\text{-}2\;0\;\text{-}1\;0] \rightarrow \mathbf{C}_4 = [\,0\;0\;0\;0\;1\;0\;0\;0\;1\;0]$,
$\mathbf{C}_4\,\mathbf{E}_D = [\,2\;\text{-}1\;0\;\text{-}1\;2\;0\;\text{-}1\;0\;\text{-}2\;1] \rightarrow \mathbf{C}_1 = [\,0\;0\;0\;0\;1\;0\;0\;0\;1\;1]$,

Flight causes hunger and fatigue($\mathbf{C}_2$). The dolphin herd stops and rests staying close together($\mathbf{C}_3$,$\mathbf{C}_4$). If the threat persists, they again try to flee($\mathbf{C}_1$). They do not search for food or eat during this sequence of events since survival is most important. This resonant limit cycle shows a "hidden" pattern[8] in the causal structure of the virtual world.

3. Combined Fuzzy Cognitive Maps

FCM matrices additively combine to form new FCMs[8]. This allows combination of FCMs for different actors or environments in the virtual world. The new (augmented) FCM includes the union of the causal concepts for all the actors and the environment in the virtual world. If a FCM does not include a concept the rows and columns corresponding to the edges into and out of that concept equal zero. The sum of the augmented (zero-padded) FCM matrices for each actor forms the virtual world:

$$\mathbf{F} = \sum_{i=1}^{n} w_i \mathbf{F}_i \tag{2}$$

where w_i are positive weights for the ith FCM $\mathbf{F}_i$. The weights state the relative value of each FCM in the virtual world. We can ask experts for the weights or learn them from data with neural learning laws.

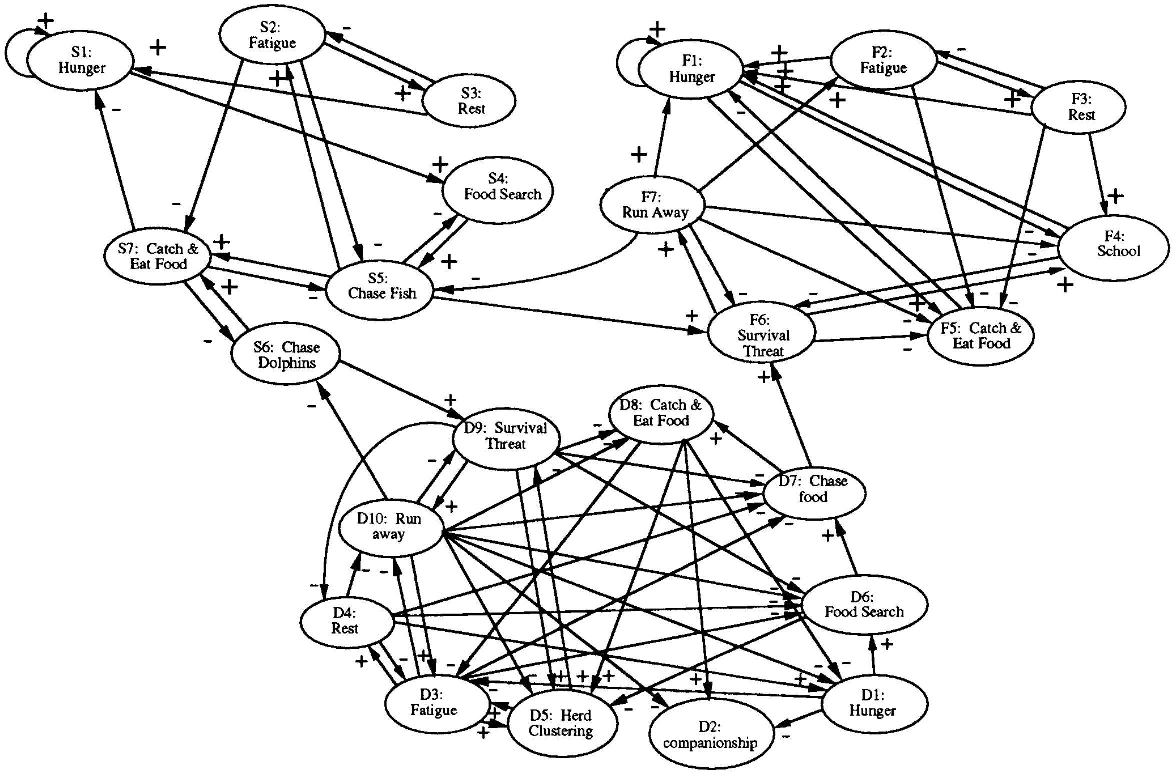

Figure 2. Augmented FCM for different actors in a virtual world. The actors interact through linked common causal concepts such as chasing food and avoiding a threat.

Figure 2 shows three FCMs for an undersea virtual world. There are fish, shark, and dolphin actors. The common causal concepts are survival threats and feeding. This results in threats to the fish or to the dolphins. This FCM leads to chases, escapes, and other interactions between the actors. A hungry shark chasing the dolphins leads to the limit cycle in section 2. Augmenting the FCM matrices gives a large but sparse FCM since the actors respond to each other in few ways.

4. FCMs in Virtual Worlds

FCMs endow virtual worlds with goals and intentions as they define dynamic physical and social environments. This gives the "common representation" needed for a virtual world[1]. We can combine many simple actions to model "intelligent" behavior [2,3]. Complex actions such as walking emerge from networks of simple reflexes. Simple FCMs mimic this process as finite state machines with binary limit cycles.

The output of a simple FCM is a resonant binary limit cycle that describes simple actions or goals. This output can control smaller FCMs or fuzzy control systems. These systems can drive visual, auditory, or tactile outputs of the virtual world. The FCM can control the temporal associations or timing cycles that structure virtual worlds.

Augmented FCMs can drive the actors in a virtual world. The FCM state vector drives the motion of each character. Each output turns a function on or off [2]. Linear equations of motion drive each actor and compute its motions between the states. The result is the complex dance between the actors moving in a two-dimensional ocean in Figure 3. The forcing function

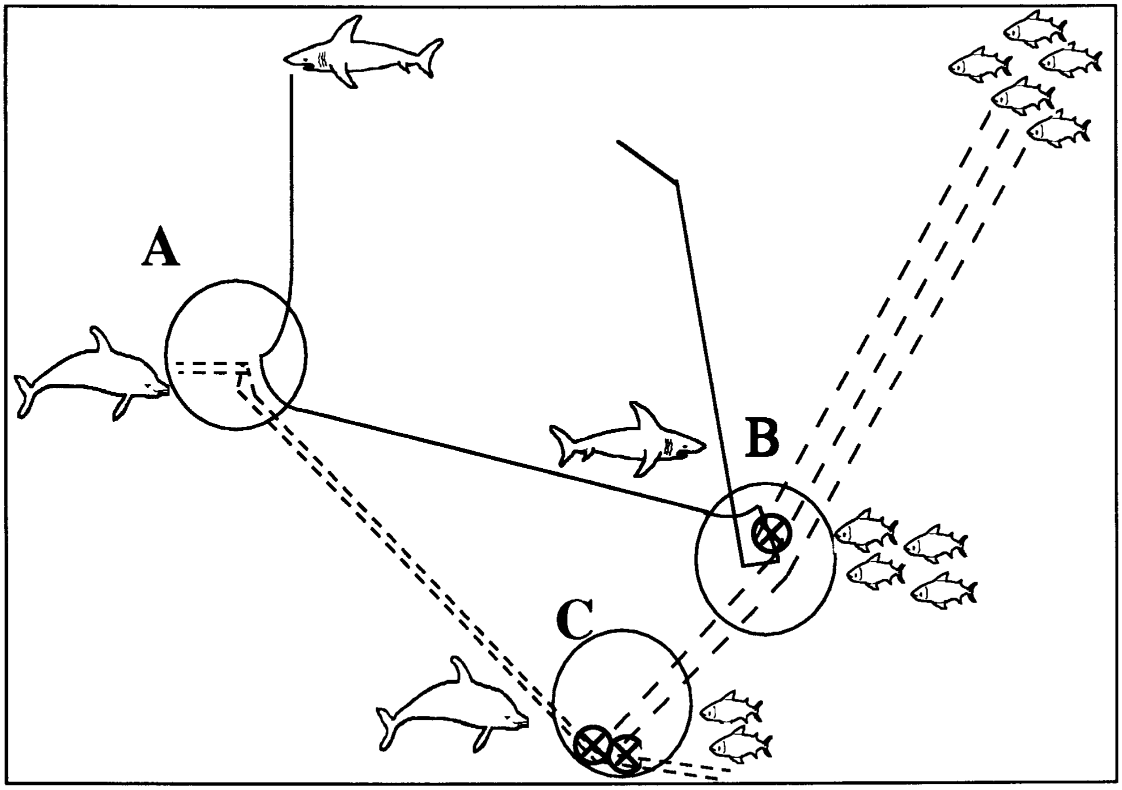

Figure 3. FCMs control the virtual world. The augmented FCM controls the actions of the actors. In event A the hungry shark forces the dolphin herd to run away. Each dashed line stands for a dolphin swim path. In event B the shark finds the fish and eats some. Each dashed line stands for the path of a fish in the school. The cross shows the shark eating a fish. In event C the hapless fish run into the hungry dolphins and suffer more losses. The solid lines are the dolphin paths and the dashes are the fish swim paths. The cross shows a dolphin eating a fish.

is a hungry shark. The shark meets the dolphins who cluster and then flee the shark. The shark chases but cannot keep up. The shark continues its search for food and finds the fish. It eats the slower fish and then, with hunger sated, it rests. Meanwhile the hungry dolphins search for food and eat more fish. Each actor responds to the actions of the other actors. The FCM blends concepts such as hunger with actions such as search for food.

5 Adaptive Fuzzy Cognitive Maps

Simple FCMs are matrix temporal associative memories[8] that associate causal events in limit cycles. Differential Hebbian learning encodes how changes in a concept map to changes in another concept. Causal learning laws infer causal links from sample data.

The differential Hebbian learning law [8] correlates changes in causal concepts

$$\dot{e}_{ij} = -e_{ij} + \dot{C}_i(x_i)\dot{C}_j(y_j) \tag{3}$$

The *discrete* change ΔC_i lies in [-1,1]. So $\Delta C_i \Delta C_j > 0$ iff concepts C_i and C_j move in the same direction. $\Delta C_i \Delta C_j < 0$ iff concepts C_i and C_j move in opposite directions. So (3) tends to learn patterns of causal change. The discrete update equation for differential Hebbian learning is[9]:

$$e_{ij}(t+1) = \begin{cases} e_{ij}(t) + c_t\left[\Delta C_i(x_i)\Delta C_j(y_j) - e_{ij}(t)\right] & \text{if } \Delta C_i(x_i) \neq 0 \\ e_{ij}(t) & \text{if } \Delta C_i(x_i) = 0 \end{cases} \tag{4}$$

where c_t is a decreasing learning coefficient. The weight matrix updates only when a causal change occurs at the input. Else the edge "forgets" the causal inferences as the signal exponentially decays.

Differential Hebbian learning encoded a feeding sequence and a chase sequence in the adjacency matrix. The neurons in the ith row learn only when $\Delta C_i(x_i)$ equals 1 or -1. The resulting adjacency matrix is:

$$\mathbf{E}_D = \begin{array}{c|cccccccccc|} & D_1 & D_2 & D_3 & D_4 & D_5 & D_6 & D_7 & D_8 & D_9 & D_{10} \\ \hline D_1 & -0.4 & 0.0 & -0.2 & 0.0 & 0.2 & 0.8 & -0.4 & 0.0 & 0.0 & 0.2 \\ D_2 & 0.0 & -0.5 & 0.5 & -0.5 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ D_3 & -0.5 & 0.0 & -0.5 & 1.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ D_4 & 1.0 & 0.0 & 0.0 & -0.7 & -0.7 & -0.3 & 0.0 & 0.0 & 0.0 & 0.0 \\ D_5 & 0.4 & -0.2 & 0.5 & -0.6 & -0.9 & -0.1 & 0.0 & 0.0 & 0.0 & -0.1 \\ D_6 & 0.0 & 0.0 & -0.2 & 0.0 & 0.2 & -0.4 & 0.8 & -0.4 & 0.0 & 0.2 \\ D_7 & 0.0 & -0.4 & -0.2 & 0.0 & 0.2 & 0.0 & -0.4 & 0.8 & 0.0 & 0.2 \\ D_8 & 0.0 & 0.6 & -0.2 & 0.0 & 0.6 & 0.0 & 0.0 & -0.4 & 0.0 & 0.2 \\ D_9 & -0.5 & 0.0 & -0.5 & 0.0 & 1.0 & -0.5 & -0.5 & -0.5 & -0.5 & 1.0 \\ D_{10} & 0.0 & 0.0 & 1.0 & -0.5 & -1.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.5 \\ \hline \end{array}$$

This learned edge matrix resembles the FCM in Figure 1. The causal conditions that it lacks were not in the training set. The diagonal of the matrix contains terms for self-inhibition of each node. This occurs since each node is on for one cycle before the matrix transitions to the next state.

$$C_0 = [\,0\;0\;0\;0\;0\;0\;0\;0\;1\;0\,]$$

$$\begin{aligned}
C_0\,\mathbf{E}_D &= [-0.5\;\;0.0\;-0.5\;\;0.0\;\;1.0\;-0.5\;-0.5\;-0.5\;-0.5\;\;1.0] \rightarrow C_1 = [\,0\;0\;0\;0\;1\;0\;0\;0\;0\;1\,], \\
C_1\,\mathbf{E}_D &= [\;0.4\;-0.2\;\;1.5\;-1.1\;-1.9\;-0.1\;\;0.0\;\;0.0\;\;0.0\;-0.6] \rightarrow C_2 = [\,0\;0\;1\;0\;0\;0\;0\;0\;0\;0\,], \\
C_2\,\mathbf{E}_D &= [-0.5\;\;0.0\;-0.5\;\;1.0\;\;1.0\;\;0.0\;\;0.0\;\;0.0\;\;0.0\;\;0.0] \rightarrow C_3 = [\,0\;0\;0\;1\;1\;0\;0\;0\;0\;0\,], \\
C_3\,\mathbf{E}_D &= [\;1.4\;-0.2\;\;0.5\;-1.3\;-1.5\;-0.5\;\;0.0\;\;0.0\;\;0.0\;\;0.0] \rightarrow C_4 = [\,1\;0\;0\;0\;0\;0\;0\;0\;0\;0\,], \\
C_4\,\mathbf{E}_D &= [-0.4\;\;0.0\;-0.2\;\;0.0\;\;0.2\;\;0.8\;-0.4\;\;0.0\;\;0.0\;\;0.2] \rightarrow C_5 = [\,0\;0\;0\;0\;0\;1\;0\;0\;0\;0\,], \\
C_5\,\mathbf{E}_D &= [\;0.0\;\;0.0\;-0.2\;\;0.0\;\;0.2\;-0.4\;\;0.8\;-0.4\;\;0.0\;\;0.2] \rightarrow C_6 = [\,0\;0\;0\;0\;0\;0\;1\;0\;0\;0\,], \\
C_6\,\mathbf{E}_D &= [\;0.0\;-0.4\;-0.2\;\;0.0\;\;0.2\;\;0.0\;-0.4\;\;0.8\;\;0.0\;\;0.2] \rightarrow C_7 = [\,0\;0\;0\;0\;0\;0\;0\;1\;0\;0\,], \\
C_7\,\mathbf{E}_D &= [\;0.0\;\;0.6\;-0.2\;\;0.0\;\;0.6\;\;0.0\;\;0.0\;-0.4\;\;0.0\;\;0.2] \rightarrow C_8 = [\,0\;1\;0\;0\;1\;0\;0\;0\;0\;0\,], \\
C_8\,\mathbf{E}_D &= [\;0.4\;-0.2\;\;1.1\;-1.1\;-0.9\;-0.1\;\;0.0\;\;0.0\;\;0.0\;-0.1] \rightarrow C_2 = [\,0\;0\;1\;0\;0\;0\;0\;0\;0\;0\,].
\end{aligned}$$

This is the chase, rest, eat, and rest cycle from before.

In differential Hebbian learning a FCM learns from actions as it correlates changes. This lets the actors in a virtual world learn from experience. The learning law in (4) can learn higher order causality if it correlates multiple causes with an effect.

5. Conclusions

FCMs provide a simple structure for a virtual world. They define the causal links between events and give the FCM an "arrow in time." FCMs apply to data visualization as well. They show how variables relate to one another and cause changes. This applies to problems in economics, medicine, and politics[11] where the social and causal structure can change in complex ways that may amount to no more than changing a sign or a magnitude in a FCM.

Adaptive FCMs that learn with neural-like laws link causes and actions based on training data from a real-world system or from a human expert. This can allow a virtual world to learn how an expert chooses winning stocks or top wines or the best way to avoid hungry sharks.

6. References

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